DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

37. Proposed by A. H. BELL, Box 184, Hillsboro, Illinois.

Find the first four integral values of n in $\dfrac{n(5n-3)}{2}=\square$.

I. Solution by the PROPOSER, and Prof. J. SCHEFFER, A. M., Hagerstown, Maryland.

Let the heptagonal numbers $\dfrac{n(5n-3)}{2}=\square=y^2$. Clearing of fractions, then multiplying by 20 and adding 9 to both sides, $(10n-3)^2=40y^2+9=\square=x^2$. $\therefore n=(x+3)/10$........(1). Let $x^2-40y^2=9$ be written $3^2x_1{}^2-40.3^2y_1{}^2=3^2$. Dividing by 3^2 and solving $x_1{}^2-40y_1{}^2=1$, the convergent of $\sqrt{40}$ is $19/3$. $\therefore x_1=19$; by the general formula $x_{n+1}=2x_1\times x_n-x_{n-1}$, we have $x_1=1$, 19, 721, 27379, 1039681, 39, 480, 499, etc. As $x=3x_1$ and as integral values for n can only be obtained by the numbers ending in 9, then in (1) $n=1$, 6, 8214, and 11844150.

II. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

The expression readily reduces to $10n^2-6n=\square$......(1). It is readily seen that $n=1$ satisfies this equation. Take $n=m+1$, substitute it in (1), reduce and we have $10m^2+14m+4=\square=$(say) $(pm-2)^2$, from which we obtain $m=(4p+14)/(p^2-10)$. Take $p=4$ and we have $m=5$, and $n=6$, the second value. Now take $n=m+6$, substitute in (1) and reduce as before and we find, $m=43$, and $n=49$, the third value. In $(4p+14)/(p^2-10)$ take $p=19/6$, $p^2-10=1/36$ and we have $m=960$, and $n=961$, the fourth value.

III. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

If we put the expression equal x^2 and reduce, we readily obtain $10n=3\pm\sqrt{40x^2+9}$. Putting $x=1$, 2, 9, 40 and 77, respectively, I find the first four integral values of n to be, respectively, ±1, 6, -25, and 49.

38. Proposed by H. C. WILKES, Skull Run, West Virginia.

Let n be any number and let $n^3+1=x$.
Then $x^3+(2x-3)^3+(nx-3n)^3=n^3x^3$. Demonstrate.

I. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.

The simplest way is to substitute the value of x and expand. An identity is the result.

II. Solution by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Substituting n^3+1 for x, $(n^3+1)^3+(2n^3-1)^3+(n^4-2n)^3=(n^4+n)^3$,